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Three-dimensional asymmetric flow through a planar diffuser: Effects of divergence angle, Reynolds number and aspect ratio ☆

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ABSTRACT

In this study, three-dimensional laminar incompressible flow through a planar diffuser (gradual expansion) for different divergence half angles of the diffuser (θ), Reynolds numbers (Re), and aspect ratios (AR) is studied numerically. The numerical model is validated against the experimental data available in the literature. The effects of θ and AR on the critical Reynolds number at which the onset of asymmetric flow is observed, are investigated.

1. Introduction

A common component of piping systems is the expansion, which can be sudden or gradual. Expansions are used to decelerate the flow and recover static pressure. Laminar flow through expansions has numerous industrial [1–4] and biomedical [5] applications, and needs to be understood in more details. One important phenomenon in laminar flow through symmetric expansions is flow asymmetry. It is believed that the cause of asymmetry lies in the instability of the shear layers formed by flow separation. The instability originates from small periodic perturbations embedded in the shear layers, amplified to form wavy flow pattern, and then vortex-like structures as they are convected downstream [6].

The asymmetry onset of laminar, incompressible flow through a sudden expansion has been investigated extensively by the use of experimental and numerical methods. These studies have shown that laminar flow in a plane symmetric sudden expansion exhibits symmetric behavior for Reynolds numbers lower than a critical value, and stable bifurcated flow regime for Reynolds numbers higher than the critical value. Results have also shown that the flow becomes unsteady as the Reynolds number continues to increase. Durst et al. [7] measured the flow downstream of a planar 3:1 symmetric sudden expansion in a duct using laser Doppler anemometry. They observed symmetric velocity profiles at a Reynolds number of $Re = 56$ ($R_e = hU_m/\nu$). At higher Reynolds numbers, i.e. 114 and 252, the flow patterns were asymmetric, but stable. Fearn et al. [8] studied the origin of steady asymmetric flows in a symmetric sudden expansion using experimental and numerical techniques. Transition from a steady symmetric flow to a steady asymmetric flow was shown to occur due to a pitchfork bifurcation at the critical Reynolds number of 80.9, for a channel expansion ratio of 3. Mizushima et al. [9] investigated the stability of a two-dimensional (2-D) flow in a symmetric channel with a suddenly expanded section using nonlinear stability theory. According to their numerical results, the flow was steady and symmetric at very low Reynolds numbers, and lost stability above a critical Reynolds number resulting in an appearance of asymmetric flow. A 2-D numerical study for bifurcation phenomena in symmetric plane sudden expansions for several Reynolds numbers and various expansion ratios was performed by Drikakis [10]. His calculations revealed that the flow was separated symmetrically up to a certain Reynolds number while, when increasing the Reynolds number symmetry-breaking bifurcation occurred and separation bubbles of different sizes formed on the upper and lower walls. The asymmetries became stronger with increasing Reynolds number. The critical Reynolds number of the symmetry-breaking bifurcation reduced with increasing the expansion ratio. Critical Reynolds numbers of the symmetry-breaking bifurcation for expansion ratios of 1:2, 1:3, 1:4, 1:5, 1:6, 1:8, and 1:10 were found to be 216, 80, 53, 41, 33, 28, and 26, respectively. Thiruvengadam et al. [11] presented the simulation results of three-dimensional (3-D) laminar forced convection in plane symmetric sudden expansion in rectangular duct with an expansion ratio of two and downstream aspect ratio of two for $Re = 225$, 300 and 400. The bifurcated flow for this Reynolds number range was laminar, steady and asymmetric in the transverse directions, but symmetric relative to the center width of the duct in the spanwise direction. Tsui et al. [12] performed 3-D computations to study the flow through a symmetric sudden expansion at low Reynolds numbers. They examined the effect of side wall on the bifurcating flow in a channel with a symmetric sudden expansion. Their results showed that the critical Reynolds number of the symmetry-breaking bifurcation increased as the aspect ratio was reduced. Numerical simulations of the 2-D laminar

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Nomenclature

μ	Shear viscosity, kg/m s
ν	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3
θ	Divergence half angle of the diffuser, degree
AR	Aspect ratio
H	The height of the downstream channel, m
h	The height of the diffuser entrance, m
H_i	The height of the inlet, m
L_1	The length of the upstream channel, m
L_2	The length of the downstream channel, m
$R_e = hU_m/\nu$	Reynolds number
t	Time, s
U_m	Maximum velocity at the diffuser entrance, m/s
W	The weight of the channel, m
x	Axial (streamwise) position, m
y	Spanwise position, m
z	Transverse position, m

flow ($100 \le Re \le 2500$) over a backward-facing step channel using two commercially-available Computational Fluid Dynamics (CFD) codes (Fluent and Comsol Multiphysics) were reported by Lima et al. [13]. The agreement between the experimental and numerical results was acceptable only in the low Reynolds number range. It was observed that differences between the experimental and numerical results increased at higher Reynolds numbers. They attributed these dissimilarities between the experimental and numerical predictions to the fact that 3-D flow features cannot be captured accurately by a 2-D model.

Unlike the asymmetric flow in sudden expansions, the flow bifurcation in gradual expansions (diffusers) has been scarcely investigated. Numerical model of laminar separated flow in symmetric 2-D planar diffusers has been investigated by Tsui et al. [14]. They observed that static pressure was recovered in the diffuser as expected, and reached a peak value somewhere downstream of the reattachment point of the large recirculating flow. The appearance of a small recirculating flow region caused the pressure recovery to exhibit periodic variation. The pressure recovery deteriorated as the diffusion angle was increased. One particular exception was for low Reynolds number ($Re = 56$) where the pressure recovery was found to increase when θ was increased from 15° to 30°. Sparrow et al. [15] investigated fluid flows in a conical diffuser by means of a 3-D numerical model. They found that symmetric flow separation occurred for a diffuser expansion angle of 5° for inlet Reynolds numbers less than about 2000. Results from the 10° and 30° simulations showed symmetric separation at all investigated Reynolds numbers (500–33,000).

The numerical model used by Tsui et al. [14] was 2-D, whereas, the flow field in a planar diffuser is 3-D in nature. A 2-D model cannot predict the nonlinear phenomena such as flow separation and bifurcation, accurately. Experimental evidence suggests that 3-D effects are significant in the development of asymmetric flows [8]. That is the reason for unsatisfactory agreement between the results of the 2-D models used by Tsui et al. [14] and Lima et al. [13] and the experimental measurements for a sudden expansion. The agreement between the degree of flow asymmetry of the experimental and 2-D numerical results was not satisfactory. It is believed that the cause of discrepancy is mainly due to the 3-D flow effects in the regions of recirculation which cannot be modeled accurately by a 2-D model.

As the literature review indicates, most of the studies on asymmetric flow have been performed in sudden expansions and the studies on asymmetric flow in planar diffusers have been performed in 2-D domains. In the present study, a 3-D experimentally-validated numerical model is proposed to study the onset of flow asymmetry in a planar diffuser. The effects of divergence angle, Reynolds number and aspect ratio on the flow asymmetry are analyzed.

2. Numerical models

The 3-D computational domain used in this study is shown in Fig. 1. A constant velocity is used as an inlet boundary condition. A fully developed channel flow is developed at the diffuser entrance. Pure incompressible air at 20° ($\rho = 1.2$ kg/m^3, $\mu = 1.8 \times 10^{-5}$ kg/m s) is used as the working fluid. No-slip boundary conditions are prescribed on the walls.

The 3-D laminar incompressible governing equations are solved using the Fluent software in which a finite-volume scheme is used to discretize the governing equations. In order to improve the accuracy of the simulations, a second-order upwind scheme is used for the momentum equations. Pressure is discretized with a second-order scheme. The SIMPLEC algorithm is used for the pressure–velocity coupling. The iterations continue until the scaled residual be smaller than 10^{-11}. Because the flow field is symmetric relative to the center width of the duct in the spanwise direction, the half of the domain is used in the simulations. Grid independence tests are performed using three different grid densities to find the optimum grid density that provides grid independent results. The chosen grid densities for $\theta = 90°$, and $AR = 9$, for half of the duct width are $200 \times 50 \times 64$ for the downstream duct and $60 \times 50 \times 24$ for the upstream segment.

2.1. Validation

In order to validate the accuracy of the numerical model, comparisons are made between the numerical results and the experimental data reported by Durst et al. [7]. The experimental geometry used in [7] is considered in the simulation, which is shown in Fig. 1, with $\theta = 90°$, $h = 4$ mm, $H = 12$ mm, $W = 110$ mm, $H_i = 44$ mm, $L_1 = 76$ mm, and $L_2 = 200$ mm. Fig. 2 (bottom row) shows the numerical predictions of the normalized x-velocity (U/U_m) in $y = 0$ plane at $Re = 56$ and 252 at different locations downstream of the sudden expansion, along with the experimental measurements of Durst et al. [7]. Symmetric velocity profiles are observed at $Re = 56$, whereas, at $Re = 252$, the flow is asymmetric. As observed, the numerical results are in good quantitative and qualitative agreement with the experimental data.

In order to show the importance of using a 3-D model to accurately predict the flow bifurcation behavior, the comparisons of the normalized x-velocity of the 2-D model and experimental data of Durst et al. [7] for $Re = 56$ and 252 are also shown in Fig. 2 (top row). It is observed that the agreement between the numerical and experimental results is less satisfactory for the 2-D model than that for the 3-D model. The disagreement of the 2-D model is more significant at higher Reynolds numbers, at which the flow bifurcation occurs. The average differences between the experimental and 3-D

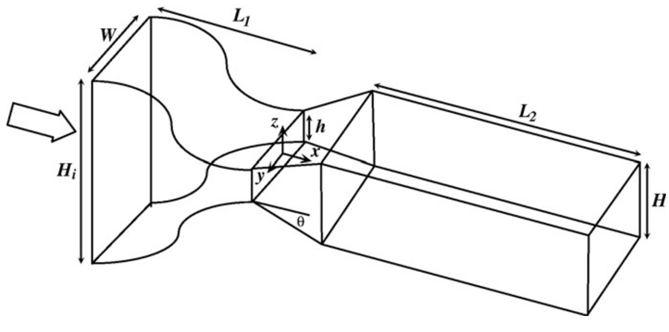

Fig. 1. The 3-D computational domain.

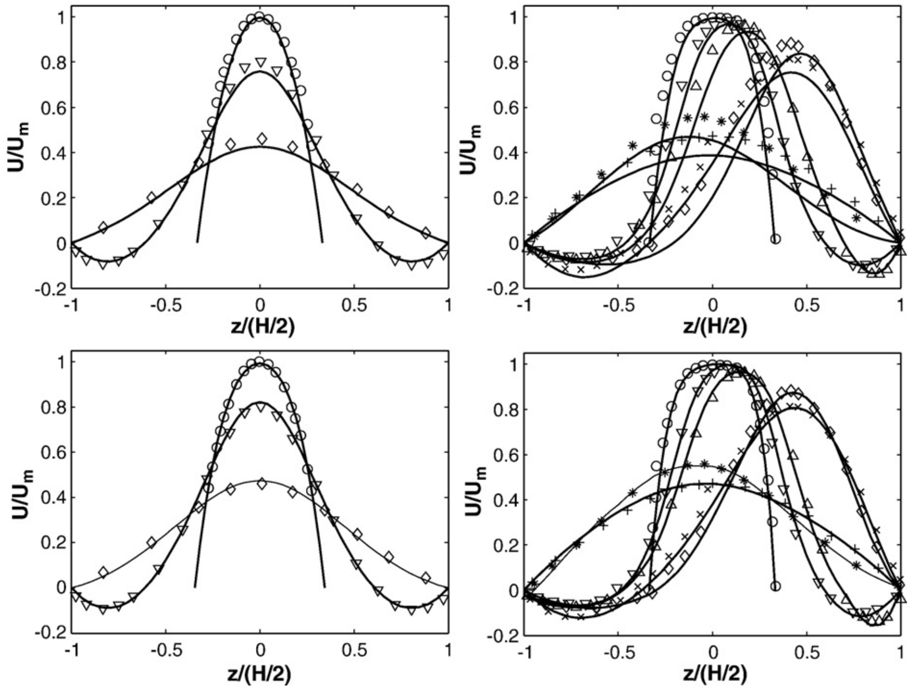

Fig. 2. Numerical results of the normalized x-velocity profiles (solid line) of the 2-D model (top row) and the 3-D model (bottom row) along with the experimental data of Durst et al. [7] (marked), at $Re = 56$, $U_m = 0.21$ m/s (left pane), and $Re = 252$, $U_m = 0.93$ m/s (right pane) at different axial locations $x/h = -0.25$ (○); 1.5 (▽); 2.5 (△); 5 (◇); 10 (×); 25 (*); 40 (+).

numerical results are 0.0095 and 0.0237, for $Re = 56$ and 252, respectively. Whereas, these numbers are 0.025 and 0.066 for the 2-D model.

In order to further emphasize the necessity of using a 3-D model to predict the flow asymmetry accurately, the x-velocity contours of the 3-D model with $AR = 9$ at $y = 0$ plane and that of the 2-D model, for $\theta = 5°$ and $Re = 280$ are shown in Fig. 3. It is observed that the 2-D model predicts asymmetric flow, whereas, the 3-D model predicts symmetric flow structure.

3. Results and discussion

The simulation results at different values of Reynolds number in the range of $60 \leq Re \leq 360$, different diffuser half angles in the range of $5° \leq \theta \leq 90°$, and two different values of aspect ratio ($AR = 4$ and 9) are presented. The simulations are performed for $h = 4$ mm, $H = 12$ mm, $W = 108$ and 48 mm, $H_i = 44$ mm, $L_1 = 76$ mm, and $L_2 = 200$ mm. Panels a and b of Fig. 4 show the values of Re vs. θ for all cases studied for the 3-D model with AR of 9 and 4, respectively. The symbols (○) and (◇) indicate the symmetric and asymmetric flow structures, respectively. Fig. 4 clearly shows that both Re and θ affect the symmetry onset of the flow structure in a planar diffuser. For every

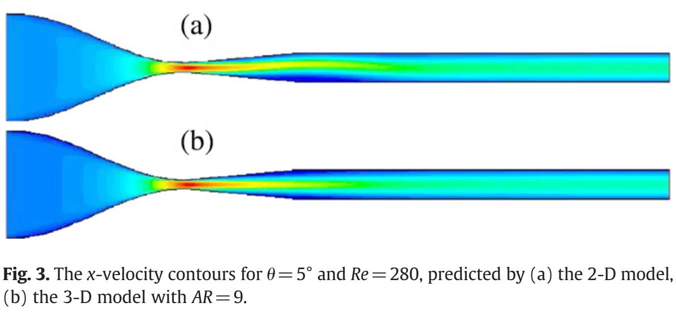

Fig. 3. The x-velocity contours for $\theta = 5°$ and $Re = 280$, predicted by (a) the 2-D model, (b) the 3-D model with $AR = 9$.

values of θ, there is a critical Re beyond which the flow is asymmetric. Increasing θ decreases the critical Re.

From the results presented in Fig. 4, it can be inferred that decreasing the aspect ratio increases the minimum Reynolds number required for asymmetric flow, i.e. flow instability occurs at higher Reynolds numbers.

It is noteworthy that the flow asymmetry can be eliminated by imposing an oscillating flow on the steady flow. In order to verify the symmetry of the unsteady flow through the diffuser, the 3-D domain with $\theta = 20°$ is considered. In the unsteady solver, a pressure-implicit with splitting of operators (PISO) is used as the pressure–velocity coupling scheme, which is recommended for transient calculations. Pressure is discretized with a pressure-based segregated solver only (PRESTO) scheme. For a constant inlet velocity of 0.08 m/s, corresponding to $Re = 300$, the flow is asymmetric as shown in Fig. 5(a). For a pulsating inlet velocity of $0.08 + 0.02\sin(20\pi t)$ m/s, the flow is symmetric at all phases of the inlet velocity as seen in Fig. 5(b), which shows the x-velocity contour at the phase of $\pi/2$. In pulsating flow regime, the oscillation period is too short to allow the instability of the shear layers to develop, and asymmetric flow separation to occur. The effects of the oscillating amplitude and frequency on the flow asymmetry need to be investigated in more details.

4. Conclusions

In this study, laminar incompressible flows through a planar diffuser for different values of diffuser angle, Reynolds number, and aspect ratio have been numerically studied in three dimensions. It was observed that for every values of diffuser angle, there is a critical Reynolds number beyond which the flow is asymmetric. Increasing diffuser angle decreases the critical Reynolds number. The critical Reynolds number of the symmetry-breaking bifurcation in the planar diffuser increases as the aspect ratio is decreased. It is possible to eliminate the flow asymmetry by imposing an oscillating flow on the steady flow.

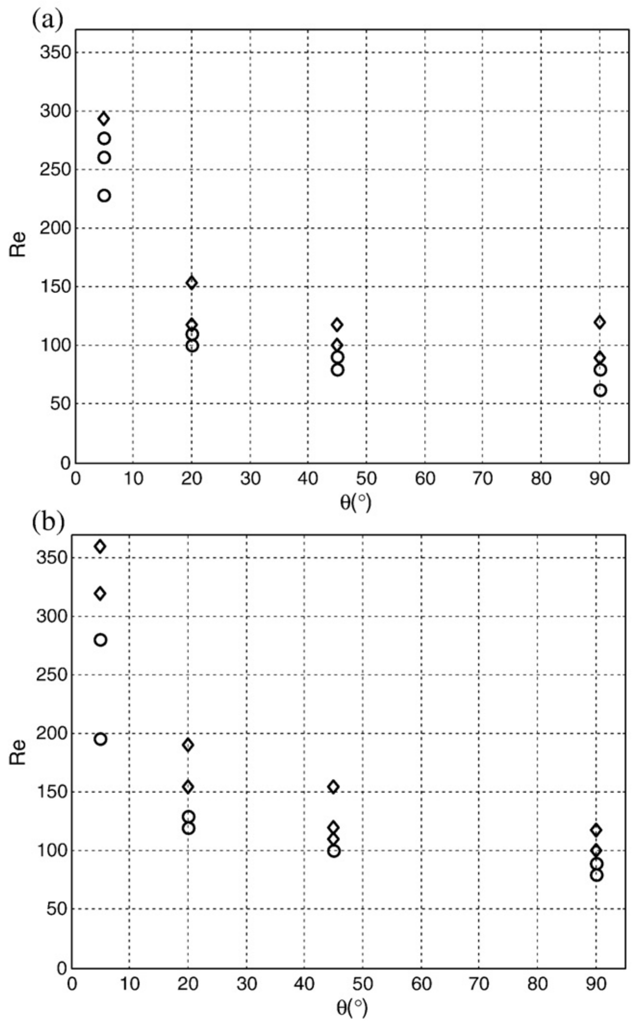

Fig. 4. *Re* vs. θ for all cases studied for (a) $AR=9$, and (b) $AR=4$. ○, symmetric flow; ◇ asymmetric flow.

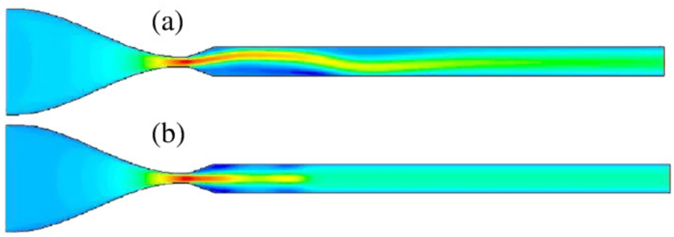

Fig. 5. The *x*-velocity contours for $\theta=20°$ predicted by the 3-D model for (a) constant inlet velocity of 0.08 m/s ($Re=300$), and (b) pulsating inlet velocity of $0.08+0.02\sin(20\pi t)$ m/s at the phase of $\pi/2$.

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